Cambridge
IGCSE

## Cambridge International Examinations

Cambridge International General Certificate of Secondary Education

## CANDIDATE

 NAMECENTRE NUMBER


CAMBRIDGE INTERNATIONAL MATHEMATICS
0607/63
Paper 6 (Extended)
October/November 2017
1 hour 30 minutes

Candidates answer on the Question Paper.
Additional Materials: Graphics Calculator

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO NOT WRITE IN ANY BARCODES.

Answer both parts A and B.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and to communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40 .

## Answer both parts A and B.

## A INVESTIGATION <br> CHEQUERED FLAGS (20 marks)

You are advised to spend no more than 45 minutes on this part.
This investigation is about the number of coloured squares on a chequered flag.


A chequered flag has two or more colours making a pattern of squares.
There is always a black square in the top left corner.

The size of a flag is $m$ by $n$, where $m$ is the number of rows and $n$ is the number of columns.

1 (a) The flag above is a 4 by 5, black and white chequered flag.

Write down the number of black squares and the number of white squares.

Black $\qquad$

White $\qquad$
(b) (i) Complete both tables.

|  | Size of flag |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 by 1 | 2 by 2 | 2 by 3 | 2 by 4 | 2 by 5 | 2 by $n$ |
| Black | 1 | 2 |  |  |  |  |
| White | 1 | 2 |  |  |  |  |


|  | Size of flag |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 4 by 1 | 4 by 2 | 4 by 3 | 4 by 4 | 4 by 5 | 4 by $n$ |
| Black | 2 |  |  |  |  |  |
| White | 2 |  |  |  |  |  |

(ii) Find two expressions, in terms of $n$, for the number of black squares and the number of white squares on a 6 by $n$ flag.

Black $\qquad$

White $\qquad$
(c) How many different sizes of black and white chequered flags are there with 12 black squares and 12 white squares?
(d) $m$ is an even number.

For the flags in this question find an expression, in terms of $m$ and $n$, for the number of black squares on an $m$ by $n$ chequered flag.

2 This is a 3 by 5 , black and white chequered flag.

(a) Write down the number of black squares and the number of white squares.

Black $\qquad$
White $\qquad$
(b) Is your expression in question 1(d) correct for this flag?

Write down the restrictions on $m$ and $n$ for your expression in question $1(\mathbf{d})$.
$\qquad$
(c) Complete both tables.

|  | Size of flag |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3 by 1 | 3 by 2 | 3 by 3 | 3 by 4 | 3 by 5 |
| Black |  |  |  |  |  |
| White |  |  |  |  |  |


|  | Size of flag |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5 by 1 | 5 by 2 | 5 by 3 | 5 by 4 | 5 by 5 |
| Black |  |  |  |  |  |
| White |  |  |  |  |  |

(d) $m$ and $n$ are both odd numbers.

Using your answer to question 1(d), find an expression, in terms of $m$ and $n$, for the number of black squares and the number of white squares.

Black $\qquad$

White $\qquad$

3 This is a 4 by 6 , black, white and grey three-coloured chequered flag.

(a) For this flag, write down the number of squares of each colour.

Black $\qquad$
White $\qquad$
Grey $\qquad$
(b) (i) Complete both tables.

|  | Size of flag |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3 by 1 | 3 by 2 | 3 by 3 | 3 by 4 | 3 by 5 | 3 by $n$ |  |
| Black |  |  |  |  |  |  |  |
| White |  |  |  |  |  |  |  |
| Grey |  |  |  |  |  |  |  |


|  | Size of flag |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 6 by 1 | 6 by 2 | 6 by 3 | 6 by 4 | 6 by 5 | 6 by $n$ |  |
| Black |  |  |  |  |  |  |  |
| White |  |  |  |  |  |  |  |
| Grey |  |  |  |  |  |  |  |

(ii) For the flags in this question, find an expression, in terms of $m$ and $n$, for the number of black squares on an $m$ by $n$ three-coloured chequered flag.
(iii) Is your expression in part (ii) correct for a flag with 2 rows? Write down the restriction on $n$ for your expression in part (ii).

4 The number of black squares on a flag with six colours is $\frac{m n}{6}$.
(a) Show that this is not true for a 16 by 14 flag with six colours.
(b) A six-coloured chequered flag has 3 black squares.

Find all the possible sizes of the flag.

5 The expression $\frac{m n}{p}$ gives the number of each coloured square on an $m$ by $n$ flag with $p$ colours. What is true about $m, n$ and $p$ ?

## B MODELLING AREAS OF POLYGONS (20 marks)

You are advised to spend no more than 45 minutes on this part.
This task looks at the relationship between the number of sides and the area of an enclosure.
A farmer has a 24 metre length of fencing to make an enclosure.
1 (a) He makes a rectangular enclosure as shown in this diagram.


NOT TO
SCALE

Complete this table to show all the possible rectangular enclosures he can make with 24 m of fencing. The sides of the enclosure are always a whole number of metres.

1 by 11 and 11 by 1 are the same rectangle.

| Width (m) | Length (m) | Area $\left(\mathrm{m}^{2}\right)$ |
| :---: | :---: | :---: |
| 1 | 11 | 11 |
| 2 | 10 | 20 |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

(b) What is the mathematical name of the rectangle that gives the maximum area of the enclosure?

2 The farmer makes an enclosure that is an equilateral triangle with a perimeter of 24 m . The perpendicular distance from a corner to the opposite side of the enclosure is $h$ metres.


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Show that the area of this triangle is $27.7 \mathrm{~m}^{2}$, correct to 1 decimal place.

3 (a) The farmer makes an enclosure that is a regular pentagon with a perimeter of 24 m .


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(i) Show that $a=72$.
(ii) The regular pentagon can be divided into 5 isosceles triangles.

The perpendicular distance from the centre of the pentagon to a side is $h$ metres.
Use trigonometry to find the value of $h$.


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(iii) Use your value of $h$ to find the area of the isosceles triangle.
(iv) Show that the area of the pentagon is $40 \mathrm{~m}^{2}$, correct to the nearest square metre.
(b) The farmer makes an enclosure in the shape of a regular hexagon with a perimeter of 24 m .

Use the method in part (a) to find the area of this hexagon.

4 The farmer makes an enclosure that is an $n$-sided regular polygon with a perimeter of 24 m . Here is an isosceles triangle from this polygon.


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(a) (i) Use this triangle and the method from question 3, to show that a model for calculating the area $\left(A \mathrm{~m}^{2}\right)$ of a regular $n$-sided polygon with perimeter 24 m is

$$
A=\frac{144}{n \tan \left(\frac{180^{\circ}}{n}\right)} .
$$

Show all your working.
(ii) Write down a condition for $n$.
(b) Use the model to show that the area of a regular octagon with a perimeter of 24 m is $43.5 \mathrm{~m}^{2}$, correct to 1 decimal place.
(c) Sketch the graph of $A=\frac{144}{n \tan \left(\frac{\left(180^{\circ}\right.}{n}\right)}$ for $3 \leqslant n \leqslant 50$ on the grid below.

(d) The farmer wants to use the 24 m of fencing to make a regular polygon with an area of $44.0 \mathrm{~m}^{2}$, correct to the nearest $0.1 \mathrm{~m}^{2}$.

Find the number of sides of this polygon.
(e) Use your graph to write down an estimate of the greatest possible area of a polygon with a perimeter of 24 m .
(f) (i) When $n$ is very large, the shape of the polygon is approximately a $\qquad$
(ii) Calculate the exact value of the greatest possible area of a shape with a perimeter of 24 m .
(g) Change the model in question 4(a) to find the area of any regular polygon with a perimeter of $P$.

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